CKM matrix unitarity and a novel type of global fits

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The aim of the paper is to propose one paradigm change of CKM global fits on experimental data from electroweak sector. The change refers to using in fits the exact unitarity constraints expressed in terms of four invariant parameters, such as moduli of the CKM matrix, and to take into account an important set of the available experimental data. In the paper we use data from nuclear beta decays, and from leptonic and semileptonic decays, in order to find the most probable numerical form of the CKM matrix, as well as the determination of decay constants, f_P, and of various form factors $f_+^{Pp}(0)$, directly from experimental results.

The consistency problem of experimental data with unitarity constraints was recently solved, and a procedure for recovering the CKM matrix elements from error affected data was provided in [1]. These unitarity constraints say that the four independent parameters s_{ij} and $\cos\delta$, with the standard notation, should take physical values, i.e. $s_{ij} \in (0,1)$ and $\cos\delta \in (-1,1)$, when they are obtained from the equations:

$$
\begin{aligned}
V_{ud}^2 &= c_{12}^2 c_{13}^2, \ V_{us}^2 = s_{12}^2 c_{13}^2, \ V_{ub}^2 = s_{13}^2 \\
V_{cb}^2 &= s_{23}^2 c_{13}^2, \ V_{tb}^2 = c_{13}^2 c_{23}^2, \\
V_{cd}^2 &= s_{12}^2 c_{23}^2 + s_{13}^2 s_{23}^2 c_{12}^2 + 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta, \\
V_{cs}^2 &= c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta, \quad (1)\\
V_{td}^2 &= s_{13}^2 c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 - 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta, \\
V_{ts}^2 &= s_{12}^2 s_{13}^2 c_{23}^2 + c_{12}^2 s_{23}^2 + 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta
\end{aligned}
$$

where $V_{ij} = |U_{ij}|$, and U_{ij} are the entries of the CKM matrix. It was shown in [1] that if the physical quantities could also depend upon CKM matrix moduli, the reconstruction of a unitary matrix from such data is essentially unique, and in the following the used independent parameters will be the moduli V_{ij}. For example, if $V_{us} = a$, $V_{ub} = b$, $V_{cd} = d$, and $V_{cb} = c$ is one set of four independent moduli, the relations (1) give the following solution

$$
s_{13} = V_{ub} = b, \ s_{12} = \frac{a}{\sqrt{1-b^2}}, \ s_{23} = \frac{c}{\sqrt{1-b^2}} \quad (2)
$$

$$
\cos\delta = \quad (3)
$$
$$
\frac{(1-b^2)(d^2(1-b^2)-a^2)+c^2(a^2+b^2(a^2+b^2-1))}{2abc\sqrt{1-a^2-b^2}\sqrt{1-b^2-c^2}}
$$

Because there are 58 groups of four independent moduli, one gets 165 different expressions for $\cos\delta$ and all have to take (roughly) the same value when computed from Eqs. (1). Thus the χ^2-function have to contain two kinds of terms: the first has to impose the fulfillment of unitarity constraints, and the second should take into account the physical quantities measured in experiments.

Concerning the experimental data we will consider data coming from super-allowed $0^+ \to 0^+$ nuclear beta decays, and from leptonic and semileptonic decays.

In the standard model the decay rate for purely leptonic decay is given by

$$
\Gamma(P \to \ell\,\overline{\nu}_\ell) = \frac{G_F^2}{8\,\pi}|U_{qq'}|^2 f_P^2 \, M_P \, m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 \quad (4)
$$

where G_F is the Fermi constant, M_P and m_ℓ are the masses of the decaying meson, and, respectively, of the final lepton, $U_{qq'}$ is the corresponding CKM matrix element, and f_P is the decay constant. In general one has also to take into account the radiative corrections which lead to a minor modification of the above formula.

The physical observable for semileptonic decays, that depends on $|U_{qq'}|$ and $f(q^2)$, is the differential decay rate which under assumption of massless leptons is written as

$$
\frac{d\,\Gamma(H \to P\,\ell\,\nu_\ell)}{dq^2} = \frac{G_F^2\,|U_{qq'}|^2}{192\pi^3 M_H^3}\lambda^{3/2}(q^2)|f(q^2)|^2 \quad (5)
$$

where $q = p_H - p_P$ is the transfered momentum, and

$$
\lambda(q^2) = (M_H^2 + M_P^2 - q^2)^2 - 4M_H^2 M_P^2 \quad (6)
$$

is the usual triangle function, and $f(q^2)$ is the global form factor which is a combination of $f_+(q^2)$ and $f_-(q^2)$. The experimenters provide numerical values for products of the form $|U_{qq'} f_+(0)|$, and in this paper we will use these numerical values. Hence from such experiments one measures, up to known factors, products of the form

$$
|U_{ij}|^2 \times f_P^2, \ \text{and/or}, \ |U_{ij}|^2 \times |f(q^2)|^2 \quad (7)
$$

It is clear that from such measurements one cannot find *two unknown* quantities, let's say, $|f(q^2)|$ and $|U_{ij}|$, if we have no supplementary constraints. Our point of view is that the unitarity constraints, which depend only on $|U_{ij}|$ moduli, see relations (2)-(3), provide the necessary tool for the separation of moduli, and $f(q^2)$, or f_P.

Before defining our type of global fit we make one natural assumption, which is: *the numerical values for all the measured moduli, $|U_{ij}|$, must be the same irrespective of the physical processes used to determine them.* The other parameters, such as the decaying constants f_P, form factors $f_+(0)$, g_A/g_V, etc., which parametrize the data from

each given experiment, are considered free parameters to be found from fit.

The first piece containing unitarity constraints entering the χ^2-function has the form

$$\chi_1^2 =$$

$$\sum_{j=u,c,t}\left(\sum_{i=d,s,b}V_{ji}^2-1\right)^2+\sum_{j=d,s,b}\left(\sum_{i=u,c,t}V_{ij}^2-1\right)^2$$
$$+\sum_{i<j}(\cos\delta^{(i)}-\cos\delta^{(j)})^2,\quad -1\le\cos\delta^{(i)}\le 1 \qquad (8)$$

and the second component, which takes into account the experimental data, is

$$\chi_2^2=\sum_i\left(\frac{d_i-\widetilde{d}_i}{\sigma_i}\right)^2 \qquad (9)$$

where d_i are the theoretical functions one wants to be found from fit, $\widetilde{d}_i$ is the numerical matrix that describes the corresponding experimental data, while σ is the matrix of errors associated to $\widetilde{d}_i$. In the following our χ^2-function will be

$$\chi^2=\chi_1^2+\chi_2^2 \qquad (10)$$

Concerning experimental data we use the following. Knowledge on $|U_{ud}|$ comes mainly from three different sources: a) super allowed, $0^+\to 0^+$, nuclear beta decays, see [2], [3], and [4], b) neutron beta decay, $n\to p\,e^+\nu$, see [5]-[13], c) and pion beta decay $\pi^+\to\pi^0e^+\nu$, [14].

The used data for the determination of the decay constants f_π, f_K, f_B, f_D, and $f_{D_s^+}$ are from the papers [15], [16], [17], [18], and, respectively, from [19]-[23]. Numerical results on $|f_+^{K\pi}(0)U_{us}|$ are from the papers [24]-[36], those upon $|f_+^{B\pi}(0)U_{ub}|$ come from [37] and [38], and the ratio $|U_{cd}f_+^{D\pi}(0)/f_+^{DK}(0)\,U_{cs}|$ is given in [39] and [40]. The papers [41]-[50] provide data on $|\mathcal{F}(1)U_{cb}|$, and [42] and [51]-[53] provide values for $|G(1)U_{cb}|$.

The central values and uncertainties used in fit are those published in the above papers, and we combined the statistical and systematic uncertainties in quadrature when experimenters provided both of them.

According to [2], the super-allowed beta decays between $T=1$ analog 0^+ states, together with the conserved vector current (CVC) hypothesis, lead to the conclusion that the ft values should be the same irrespective of the nucleus, i.e.

$$ft=\frac{K}{|G_V|^2\,|M_F|^2}=\text{const}, \qquad (11)$$

where K is the vector coupling constant for semi-leptonic weak intercations, f is the statistical rate function, and t is the partial half-life. Because the above relation is only approximately satisfied, one defines a "corrected" $\mathcal{F}t$ value, which should be "constant", as

$$\mathcal{F}t\equiv ft(1-\delta_R)(1-\delta_C)=\frac{K}{2|G_V|^2(1+\Delta_R^V)} \qquad (12)$$

where δ_C is the isospin-symmetry-breaking correction, δ_R is the transition-dependent part of the radiative correction, and Δ_R^V is the transition-independent part. Numerical values for $\mathcal{F}t$ are given in [2], [3], and [4]. In our fit we use the above formula with $|G_V|^2=|U_{ud}|^2$, by supposing that $g_V(0)=1$, as CVC requires, and $|U_{ud}|$ and Δ_R^V are the free parameters to be obtained from fit. Similarly for the neutron beta decay data we make use of the formula

$$|U_{ud}|^2(1+3\lambda^2)=\frac{4908.7(1.9)\,s}{\tau_n} \qquad (13)$$

see [54], where τ_n is the neutron mean life, and the free parameters are $|U_{ud}|$ and $\lambda=g_A/g_V$.

If one or more of the above parameters could be measured in other experiments, this approach allows us to take the results of these measurements into account. That is the case of the ratio g_A/g_V which enters in the measured asymmetry parameter A_0, see papers [55]-[58]. Their effect was a lowering of λ to the value given in the Table, while by using only results from neutron beta decay data the value, $\lambda=-1.27092\pm0.00394$, is obtained.

Values and corresponding uncertainties obtained from the fit are given in Table.

The surprising result of our fit was that Δ_R^V is not transition-independent as it is usually assumed, see Refs. [2]-[4], and [59]-[60]. For example, if one uses the data on nuclear beta decays from [4], the Δ_R^V variation is from 2.193% for ^{22}Mg, to 2.579% for ^{54}Co nucleus. For this case the corresponding mean value and uncertainty are given in Table.

If one makes use of Savard et al. data, [3], one gets

$$\Delta_R^V=(2.294\pm0.131)\% \qquad (14)$$

and the values spreading is between 2.027% for ^{74}Rb, and 2.429% for ^{34}Cl.

Our approach allows the use of all the seventeen values from [2], and one gets $\Delta_R^V=(2.362\pm2.133)\%$, where the value of Δ_R^V for ^{42}Ti provided by the fit is negative $\Delta_R^V=-4.673\%$! However the result makes sense since the corresponding $\mathcal{F}t$ is 3300 ± 1100 which is far away from the mean value given in [2] which is around 3072. Hence the fit suggests us to throw out this value. By excluding also the ^{18}Ne and ^{30}S data, that lead to greater values than the mean by a factor of 3, and, respectively of 2, one obtains

$$\Delta_R^V=(2.364\pm0.182)\% \qquad (15)$$

Looking at the three Δ_R^V values, that from the Table, and those obtained by using data from [3], Eq. (14), and, respectively, from [2], Eq. (15), one observes that they are compatible within the errors, and the better data are those coming from Ref. [3]. In all these three cases the errors provided by the fit are bigger than the theoretical estimates. We remind that the theoretical estimates are

$$\Delta_R^V(old)=(2.40\pm0.08)\%,\quad\text{and} \qquad (16)$$
$$\Delta_R^V(new)=(2.361\pm0.038)\% \qquad (17)$$

given respectively, in Ref. [60], and [4].

Parameters	Central Values and Errors
V_{ud}	$0.974022 \pm 3.9 \times 10^{-6}$
V_{us}	$0.226424 \pm 3.9 \times 10^{-6}$
V_{ub}	$(3.57604 \pm 0.00002) \times 10^{-3}$
V_{cd}	$0.226261 \pm 3.9 \times 10^{-6}$
V_{cs}	$0.973324 \pm 4.1 \times 10^{-6}$
V_{cb}	$(38.0239 \pm 0.0002) \times 10^{-3}$
V_{td}	$(9.28657 \pm 0.000035) \times 10^{-3}$
V_{ts}	$(37.0454 \pm 0.0002) \times 10^{-3}$
V_{tb}	$0.999270 \pm 2.2 \times 10^{-7}$
Δ_R^V	$(2.399 \pm 0.108)\%$
g_A/g_V	-1.26924 ± 0.00510
δ_c	$(3.104 \pm 0.096)\%$
f_π	130.784 ± 1.323
f_K	154.535 ± 1.990
f_K/f_π	1.1816 ± 0.0272
f_B	281.97 ± 0.39
f_{D^+}	220.1 ± 0.8
$f_{D_s^+}$	268.42 ± 11.22
$f_+^{K\pi}(0)$	956.8 ± 9.1
$f_+^{B\pi}(0)$	243.2 ± 20.8
$f_+^{D\pi}(0)/f_+^{DK}(0)$	0.833 ± 0.006
$\mathcal{F}(1)$	941.9 ± 78.7
$G(1)$	948.1 ± 149.8

As one conclusion one can say that our approach does not confirm the (approximate) constancy of Δ_R^V, and at the same time it allows a fine structure analysis of all nuclear beta decays, or, more precisely, of the present procedure for getting a "constant" $\mathcal{F}t$. The solving of the constancy problem of Δ_R^V could require new ideas. One suggestion could be the use of the present approach, but now with a few more steps. One can take for a "constant" Δ_R^V the value given by Eq. (14), as being the best one from all the three, and define a new $\mathcal{F}t_{new}$ as

$$\mathcal{F}t_{new} \equiv ft(1 + \Delta_R^V) = \frac{K}{2|G_V|^2(1 - \delta_R)(1 - \delta_C)} \quad (18)$$

and try to obtain from fit values for δ_R and δ_C , which can be compared with the values computed in [4]. A careful analysis of all these numerical values could say that the values obtained from fit for δ_R and δ_C are acceptable, and in such a case the problem is closed, or that they are not compatible with the theoretical knowledge on $T = 1$ analog 0^+ states. In the last case one could think to small contributions due to scalar and tensor terms in the

weak interaction model, or that $g_V(0)$ has a small nucleus dependence.

We also did a fit by using directly the values for ft given in TABLE IX from [2], such that finally we obtained four slightly different matrices for the moduli V_{ij}, which were used to get a mean value matrix and its corresponding uncertainty matrix. For that we used the natural embedment of unitary matrices into the double stochastic set, see [1], or [61], for details, and the obtained numerical values for moduli V_{ij} and their uncertainties are those given in Table.

Our result on the parameter g_A/g_V was obtained by using practically all the measurements where it is involved. The values spreading is between 1.25949 corresponding to A_0 value from paper [57], and 1.27798 obtained by using neutron lifetime from [12], such that the "unexpected" Serebrov et al. result, [5], enters naturally in the game.

A second conclusion of this approach is that it allows a "fine structure analysis" of all experiments measuring one definite physical quantity, such as Δ_R^V, or λ, as above, providing to each experimental group one measure of how far from the ideal situation their measured values are standing.

From fit we have obtained also an experimental value on δ_c, which represents the combined radiative and short-range physics corrections, see [14].

The obtained central values for the decay constants and form factors are those normally expected, all the numbers are given in MeV, and the errors are at 1σ level. The results provided for f_π and f_K are slightly different from that given in [15], in particular the lower value for f_K. The most critical situation is that for $\mathcal{F}(1)$ and $G(1)$, whose central values are almost the same by taking into account the huge errors. We remind that we used the published results, making no scaling as it is usually done, see, e.g., [62]. The simplest situation is that of $G(1)$ where the older data, [41] and [51], give a lower value, $G(1) = 808.7 \pm 77.6$, while the new ones, [52] and [53], provide $G(1) = 1,087.4 \pm 6.6$, which explain the huge error. The more complicated case is that of $\mathcal{F}(1)$ because of the experimental difficulty to measure the product $\mathcal{F}(1)|U_{cb}|$. For example, by selecting the highest, and respectively, the lowest values for $\mathcal{F}(1)$, obtained from data coming from [47], and respectively [41], their ratio is 1.37, which is too big in our opinion. As a matter of fact the highest value equals $1,133.5$, which is closer to the second value for $G(1)$ given above. Another source of error are the assumptions made on form factor parameters used to analyse the data, and on the constraints imposed on the shape of these form factors. Of course one can exclude some data, but up to now there is no definite, or accepted by all, procedure to do it.

However these problems can be solved in our approach since the important measurable quantities from experiment are $|\mathcal{F}^2(\omega_i) U_{cb}|^2|$, and these can be used directly in our approach. This procedure can be used to all the semileptonic decays, and it will lead to very precise moduli values, and, more important, to the measurement of

form factors moduli in the physical region. It also allows a direct test of the lepton universality by putting in the fit the measured data obtained separately for electron, muon and τ, etc.

More details about our fit, as well as on other physical parameters that can be computed by using the above results, will be given elsewhere.

As one final conclusion we can say that by taking properly into account the unitarity constraints we found one tool which allows the determination of CKM matrix elemnts, and of various decay constants and form factors directly from experimental data.

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